Mathematics

HP COMPUTER CURRICULUM

Functions

STUDENT LAB BOOK

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Hewlett-Packard
Computer Curriculum Series

mathematics
STUDENT LAB BOOK

functions

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This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Mathematics Student Book was written to enrich your study of mathematics by showing you how to use a computer as a modeling device. The computer is particularly helpful in quickly performing the repetitious steps of algorithms, thus making mathematical investigations easier and more exciting. You will write computer programs which will help you to understand the major concepts involved in the study of a particular mathematical area. If you become more involved in investigating the laws of mathematics, this book will have achieved its aim.

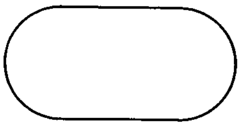
To use the Student Book for Functions, you will need the following: First, you should have one year's background in algebra. Second, the book assumes that you already know how to write a program in the BASIC programming language, and that you understand programming techniques for inputting data, performing algebraic operations, designating variables, assigning values to variables, looping, and printing results. If you do not have this background, you will want to study BASIC before attempting this material. Consult the BASIC Manual for the computer you use. Last, in order to complete the exercises in this Student Book, you will need to have access to a computer for at least two hours per week. If more time is available, you may be able to experiment further on your own, either to improve your programs or to investigate other areas of mathematics that interest you.

Each section of this book is organized in the same way. First, the mathematical concepts needed to complete the exercises are reviewed. References are listed at the end of each section in case you want to study these concepts in greater detail. Next, each exercise is presented. Finally, an approach is suggested in the Problem Analysis and a flow chart is included to illustrate this approach. The suggested approach was chosen because it brings out the concepts which are being stressed, but the program can sometimes be written more efficiently. Once you have completed the exercise by following the logic in the flow chart, you are encouraged to rewrite the program using more sophisticated programming techniques. You might also want to impose more conditions on the problem to make it more interesting to solve.

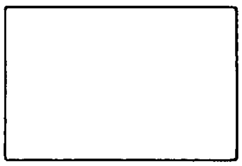
There are many different ways to solve one problem by programming. Experiment and learn as you go. You'll find you are learning something new each time, both about your subject matter and about using the computer to model mathematical algorithms.

LIST OF SYMBOLS

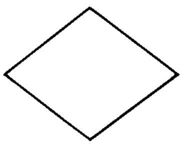
FLOW CHART SYMBOLS



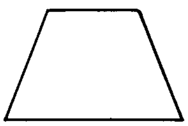
Start or Stop



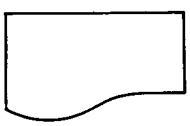
Defines a process



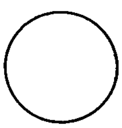
Represents a decision point



Represents computer input



Represents computer output



Used to connect one part of
a flow chart continued at
some other place

ALGEBRAIC NOTATION AND EQUIVALENT *BASIC* LANGUAGE SYMBOLS

Algebraic Notation	BASIC Notation	Meaning
+	+	Addition
–	–	Subtraction
· or X	*	Multiplication
÷ or /	/	Division
$\sqrt{X}$	SQR(X)	Square root of X
$\mid Y \mid$	ABS(Y)	Absolute value of Y
$[\![X]\!]$	INT(X)	Greatest integer less than or equal to X
=	=	Equals
≠	# or <>	Does not equal
<	<	Less than
>	>	Greater than
≤	<=	Less than or equal to
≥	>=	Greater than or equal to
	=	Replaced by
( ) or [ ]	( ) or [ ]	Inclusive brackets or parentheses
A_i	A(I)	Subscripted variable
$A_{i,j}$	A(I, J)	Double subscripted variable
(None)	RND(X)	Assign a random number to the variable X

NUMBER SET DESIGNATIONS

N — Natural number set

W — Whole number set

I — Set of integers

Q — Rational number set

Z — Irrational number set

R — Real number set

DEFINITION OF A FUNCTION

The concept of a function is fundamental to much of the study of mathematics. You will run into it again and again as you continue your studies. Using the computer to model the definition and to study the properties of functions should help you to understand this important concept.

We will adopt the following standard definitions for our studies:

A *function* is: a set of ordered pairs (x, y) of elements (numbers, chairs, people or other objects) such that to each first element, x, there is associated a unique second element, y.

The *domain* is: the set of first elements of a function.

The *range* is: the set of second elements of a function

The idea of a function has been around a long time. Noah's ark, with a pair of each known animal, is an example of *very* early use of the function concept. The word "function" was introduced by Descartes in 1637 but he limited its meaning to describe positive integral powers of x^n. Leibniz (1646–1716) used it to mean a quantity connected with a curve, such as the slope of a curve. Lejeune Dirichlet (1805–1859) first formulated a definition of "function" based on a relationship between pairs of elements. His definition is still valid but much too involved for our purposes. Our definition is derived from set theory and is more easily understood.

Exercise 1 — Averaging Grades as an Example of a Function

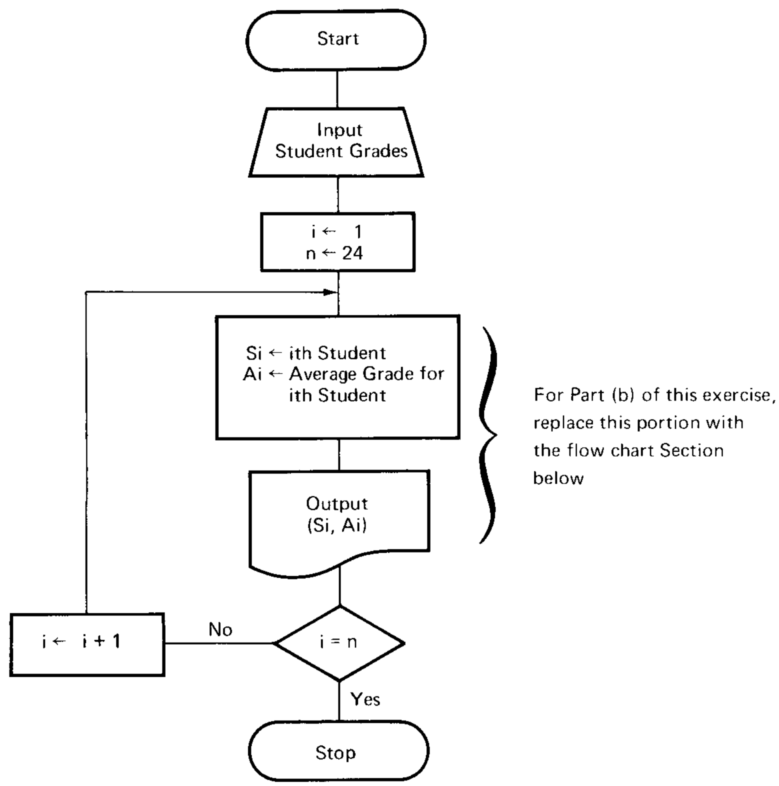
Averaging grades at the end of each grading period is a practical application of the function concept. The set of student names or numbers makes up the domain, and the set of average grades makes up the range, e.g. Student #1 is paired with the average grade of 91.

(a) *Write a computer program that will find the average of weekly grades for each student in a class of 24 students. Output the results in a set of 24 ordered pairs (student number, average grade). Use the following list of grades when running your program.*

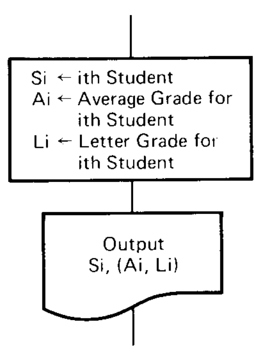
Student No.	*Grades*	*Student No.*	*Grades*
1	*100,90,98,72,96,88*	*13*	*100,65,88,72,80,80*
2	*100,95,90,100,98,100*	*14*	*85,90,88,87,86,84*
3	*60,80,62,63,76,68*	*15*	*80,70,46,55,70,64*
4	*100,70,84,90,80,80*	*16*	*75,70,58,57,70,56*
5	*100,90,92,70,88,78*	*17*	*80,65,66,66,74,60*
6	*85,65,84,70,84,74*	*18*	*100,78,73,75,60,96*
7	*100,100,94,92,94,100*	*19*	*95,55,61,60,60,100*
8	*100,95,90,87,96,100*	*20*	*90,61,57,58,60,80*
9	*100,70,73,80,88,80*	*21*	*85,85,70,80,64,80*
10	*100,75,82,87,84,88*	*22*	*80,80,40,68,84,100*
11	*100,90,88,100,94,96*	*23*	*50,84,75,84,76,85*
12	*70,75,81,80,84,80*	*24*	*50,54,68,50,85,60*

(b) Modify your program so the output will be: (student number, (numerical average, equivalent letter grade)). This output is a function in which the domain is the set of student numbers and the range is itself a set of ordered pairs. Is the relation between the numerical average and equivalent letter grade a function? Use the following grade scale in your program: A = 90-100, B = 80-89, C = 70-79, D = 60-69, F = below 60.

Flow Chart
Exercise 1



(b)



Exercise 2 — Modeling the Definition of a Function

Write a computer program that will determine if a given finite set of ordered pairs is a function. Test your program on the following sets.

(a) {(3,2) (−6,8) (3,1) (8,16) (−1/2, −2) (4,2)}

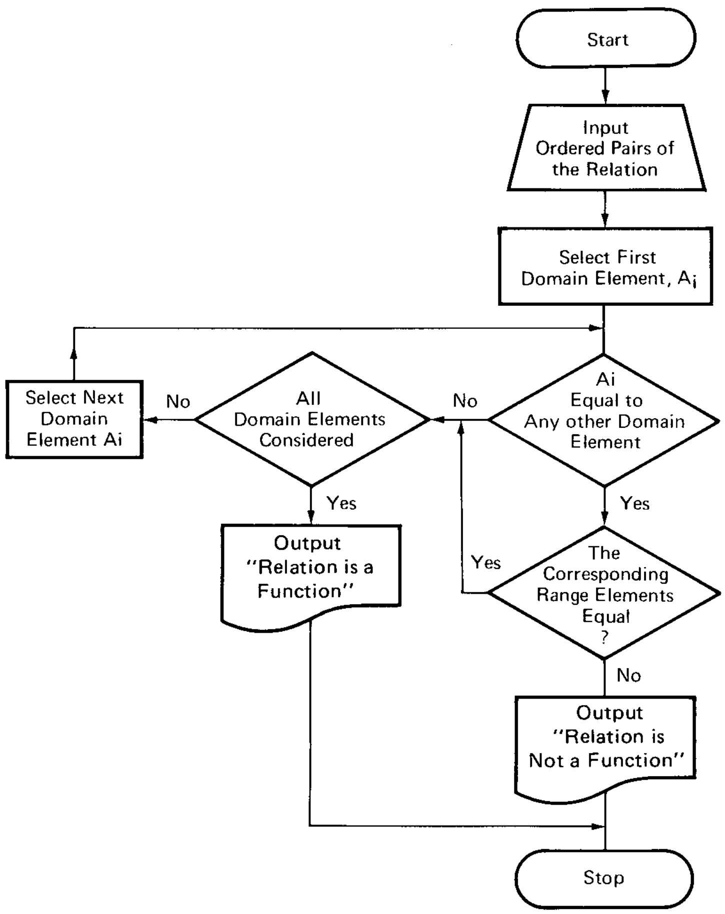
(b) {(7,1) (−8,6) (14,2) (.8,3) (6,3) (10,0) (−8, 6)}

(c) {(2,5) (−3, .75) (17, .3) (2, −.7) (3,5)}

Problem Analysis

Given a relation $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots (x_n, y_n)$ our approach will be to determine if any domain element x_i is equal to any other domain element, x_j. If not, then the relation is a function. If two domain elements are equal, then we will determine if their corresponding range elements are equal. If they are, we still have a function. Otherwise, the relation is not a function.

Flow Chart
Exercise 2



USING FINITE FUNCTIONS TO INVESTIGATE FUNCTION PROPERTIES

In this section, we will use finite functions to illustrate certain function properties. The properties discussed are also properties of infinite functions, but in order to illustrate them using the computer we need to consider each ordered pair, and must, consequently, use finite functions as our model.

THE INVERSE OF A FUNCTION

Let's look at a function A, made up of a set of ordered pairs (x,y). If we reverse the elements of each ordered pair, we have a set of ordered pairs (y,x) that we'll call B. If B is a function, it is called the *inverse* of the function A. If B is not a function, then the inverse of A does not exist.

Exercise 3 — Finding the Inverse of a Function

(a) Write a computer program to determine if a given function, f, defined by a finite set of ordered pairs, has an inverse. Use the following sets of ordered pairs in your investigation.
$f = \{(6,3)\ (8,2)\ (-1,6)\ (-.8, .2)\ (3, -7)\}$
$f = \{(7,15)\ (6,2)\ (-7,3)\ (2,1)\ (0, -3)\ (-5,10)\}$

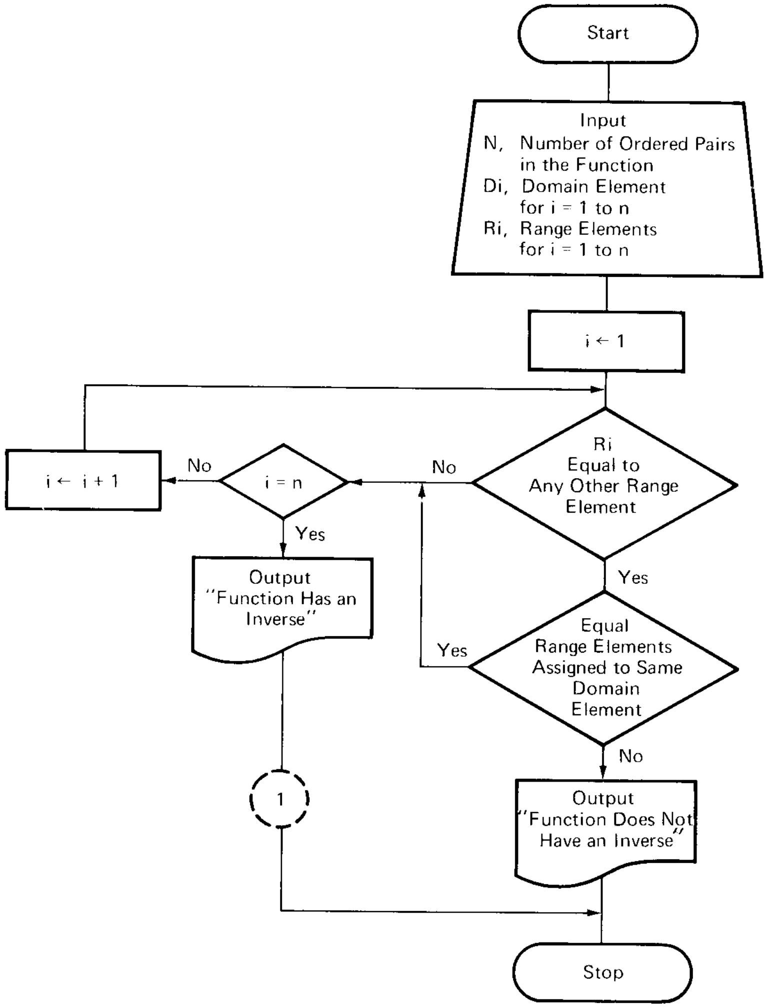
(b) Modify your program so that the inverse of each set is printed.

Problem Analysis

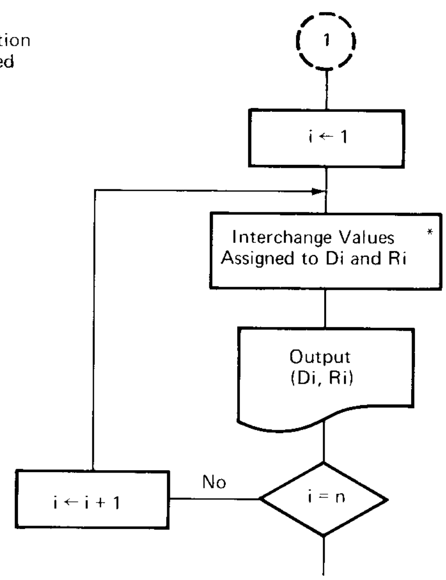
(a) To find out if the function has an inverse, we must determine if all range elements are different. If they are, the function has an inverse. If they are not different, we need to know if the equal range elements are assigned to a unique domain element. If they are, the function has an inverse. Otherwise there is no inverse.

(b) If we find that an inverse exists, the program should interchange the domain and range elements of each ordered pair in the set and print the resulting set.

Flow Chart
Exercise 3



Insert this section where indicated for part (b).



INCREASING AND DECREASING FUNCTIONS

An increasing function is defined to be a set of ordered pairs (x,y) such that if $x_1 < x_2$, then $y_1 < y_2$ for all pairs of x_1, x_2. Similarly, a decreasing function is defined so that if $x_1 < x_2$ then $y_1 > y_2$ for all pairs of x_1, x_2. A function is neither increasing nor decreasing if for $x_1 < x_2$, y_1 is consistently neither less than nor greater than y_2 for all pairs of x_1, x_2.

Before writing any programs that will identify functions as increasing or decreasing, we need to learn a technique to have the computer rearrange a set of real numbers in ascending or descending order.

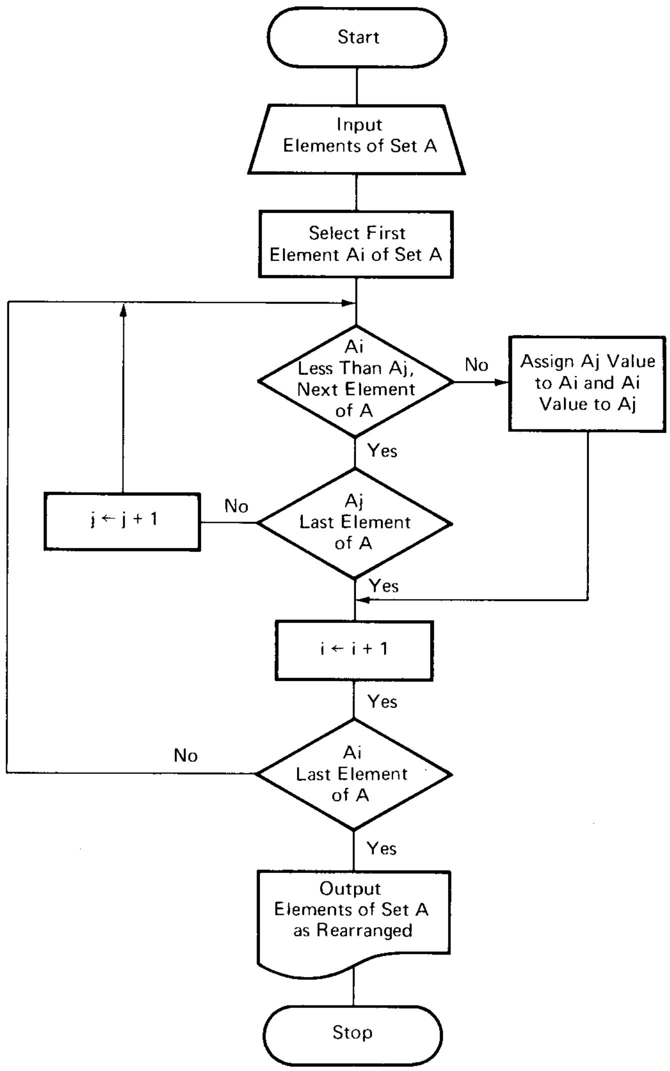
Exercise 4 — Arranging Numbers in Ascending Order

Write a computer program to arrange the elements of the set A = { 6, −2, 8, 5, 0, −8} in ascending order.

Problem Analysis

Use the process discussed below to accomplish this task. Take the first element, 6, of the set, and compare it with −2. Since 6 > −2, we interchange the positions of 6 and −2, then A = {−2, 6, 8, 5, 0, −8}. Now −2 < 6, −2 < 8, −2 < 5, −2 < 0, −2 < −8 so we interchange the positions of −2 and −8. Then A = {−8, 6, 8, 5, 0, −2}. Now we start all over again. −8 < 6, −8 < 8, −8 < 5, −8 < 0 −8 < −2, so we have the smallest number in the first position. Consider the second number 6: 6 < 8, 6 > 5, therefore we interchange positions of 6 and 5, A = {−8, 5, 8, 6, 0, −2}. 5 < 8, 5 < 6, 5 < 0. 5 > −2 so we interchange 5 and −2, A = { −8, −2, 8, 6, 0, 5, }. Now −2 is less than the succeeding numbers. Continuing the process we finally get A = { −8, −2, 0, 5, 6, 8, }.

Flow Chart
Exercise 4



Exercise 5 — Identifying Increasing and Decreasing Functions

(a) *Write a computer program to determine if a finite function is increasing, decreasing, or neither. Apply your program to the following functions and output "function is increasing," "function is decreasing," or "function is neither increasing nor decreasing" depending on your findings.*

1. $\{(.5,0), (1.75, 2), (-1, -7)\ (0, -4)\ (-1/2, -7)\ (-3, -8)\ (3, 15)\}$

2. $\{(.5, -1), (-1, 0), (7, -22), (-7, 6), (3.5, -6)\ (-5, 0)\}$

3. $\{(-6, 7), (-5, 2), (-4, 0), (-3, 2), (0, 0), (3, 1)\}$

(b) *Write a program that will test a function written in set builder notation for the increasing and decreasing properties. Apply your program to:*

1. $\{(x, f(x)) | f(x) = 6 - 2x, -5 \leq x \leq 5, x \in I\}$

2. $\{(x, f(x)) | f(x) = x^2 - x, -3 \leq x\ 3, x \in I\}$

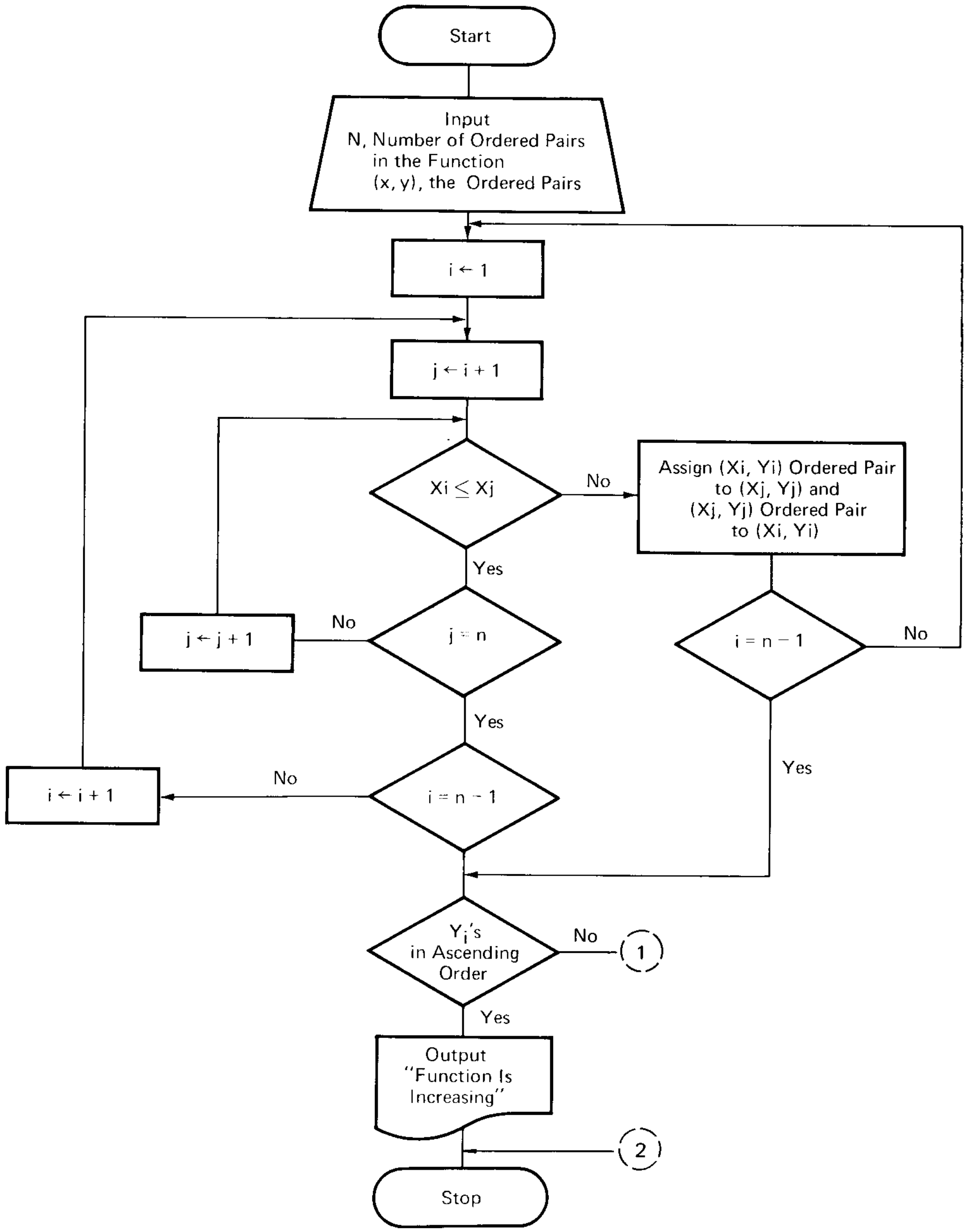
3. $\{(x, f(x)) | f(x) = x^2 - x, 0 \leq x\ 5, x \in I\}$

Problem Analysis

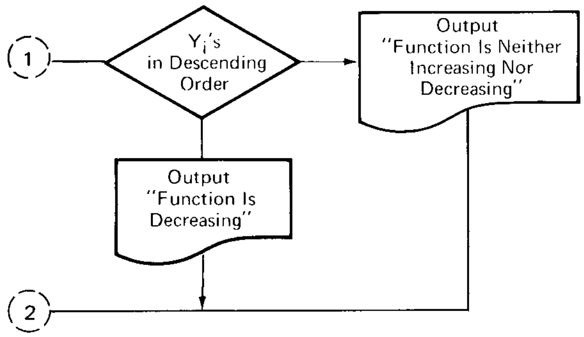
(a) Let's begin by testing the given function to see if it is an increasing function. Have the computer arrange the ordered pairs so the domain elements are in ascending order. For example, given the function $A = \{(2,0)\ (-5,1)\ (-7,0)\ (8,2)\}$, reorder it as follows: $\{(-7,0)\ (-5,1)\ (2,0)\ (8,2)\}$. Now test to see if the range elements are in ascending order. If not, the function is not increasing, in which case we must determine if the range elements are in descending order. If they are, the function is decreasing. Otherwise it is neither increasing nor decreasing.

(b) We can modify our program from (a) by merely changing the input box. First we define f(x). Then we input A equal to the lower bound of the set, and B equal to the upper bound. Finally, we set x_i equal to A, and y_i equal to $f(x_i)$. We continue this through increasing integer values of x_i until $x_i = B$.

Flow Chart
Exercise 5(a)

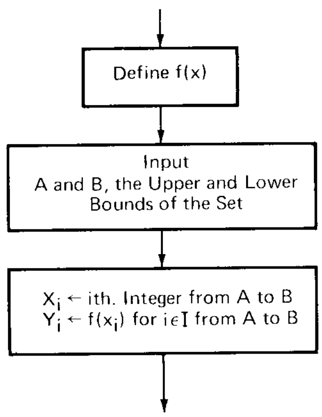


This section of the flow chart determines if the function being tested is an increasing function. The section on the next page, which connects at one and two, determines if the function is decreasing, or if it is neither increasing nor decreasing.



Flow Chart
Exercise 5(b)

Replace the Input box of the flow chart in part (a) with the following flow chart section



SYMMETRY

In the study of relations three types of symmetry are defined:

(1) *x-axis symmetry*: A set of ordered pairs is said to be symmetrical to the x-axis if for each ordered pair (a, b) there is an ordered pair (a, −b). (a, −b) is called the point in symmetry for (a, b).

(2) *y-axis symmetry*: A set of ordered pairs is said to be symmetrical to the y-axis if for each ordered pair (a, b) there is an ordered pair (−a, b).

(3) *origin symmetry*: A set of ordered pairs is said to be symmetrical to the origin if for each ordered pair (a, b) there is an ordered pair (−a, −b).

Exercise 6 — Symmetry

(a) Write a computer program that will determine if a given finite set of ordered pairs is symmetrical to the X-axis. Have the computer output which ordered pairs do not meet the conditions of symmetry. Apply your program to the following sets:

(1) {(6, 3) (−2, −1) (−2, 3) (.7, −5) (−2, 1) (.7, 5) (6, −3) (−3, −2) (−2, −3)}

(2) {(2, 15) (7, 0) (6, −1) (.5, −2) (6, 1) (2, −15) (.5, 2)}

(3) {(−5, 1) (7, 3) (6, .3) (0, 0) (0, −2) (6, −.3) (7, −3) (−5, −1) (0, 2)}

(b) Write a computer program that will determine if a given finite set of ordered pairs is symmetrical to the origin. Apply your program to the following sets:

(1) {(−7, 1) (−6, 2) (−5, 3) (−4, 4) (5, −3) (6, −2) (7, −1)}

(2) {(−7, 1) (−6, 2) (−5, 3) (−4, 4) (7, −1) (6, 2) (5, −3) (4, −4)}

(3) {(−7, 1) (−6, 2) (−5, 3) (−4, 4) (7, −1) (6, −2) (5, −3) (4, −4) (0, 0)}

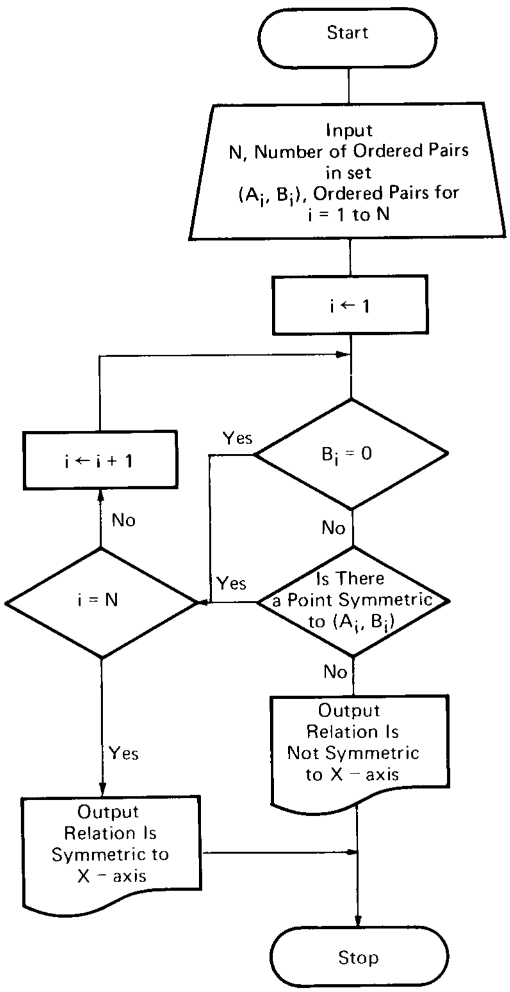
(4) {(−7, 1) (−6, 2) (−5, 3) (0, 4) (3, 1) (7, −1) (6, −2) (5, −3) (−3, −1)}

Problem Analysis

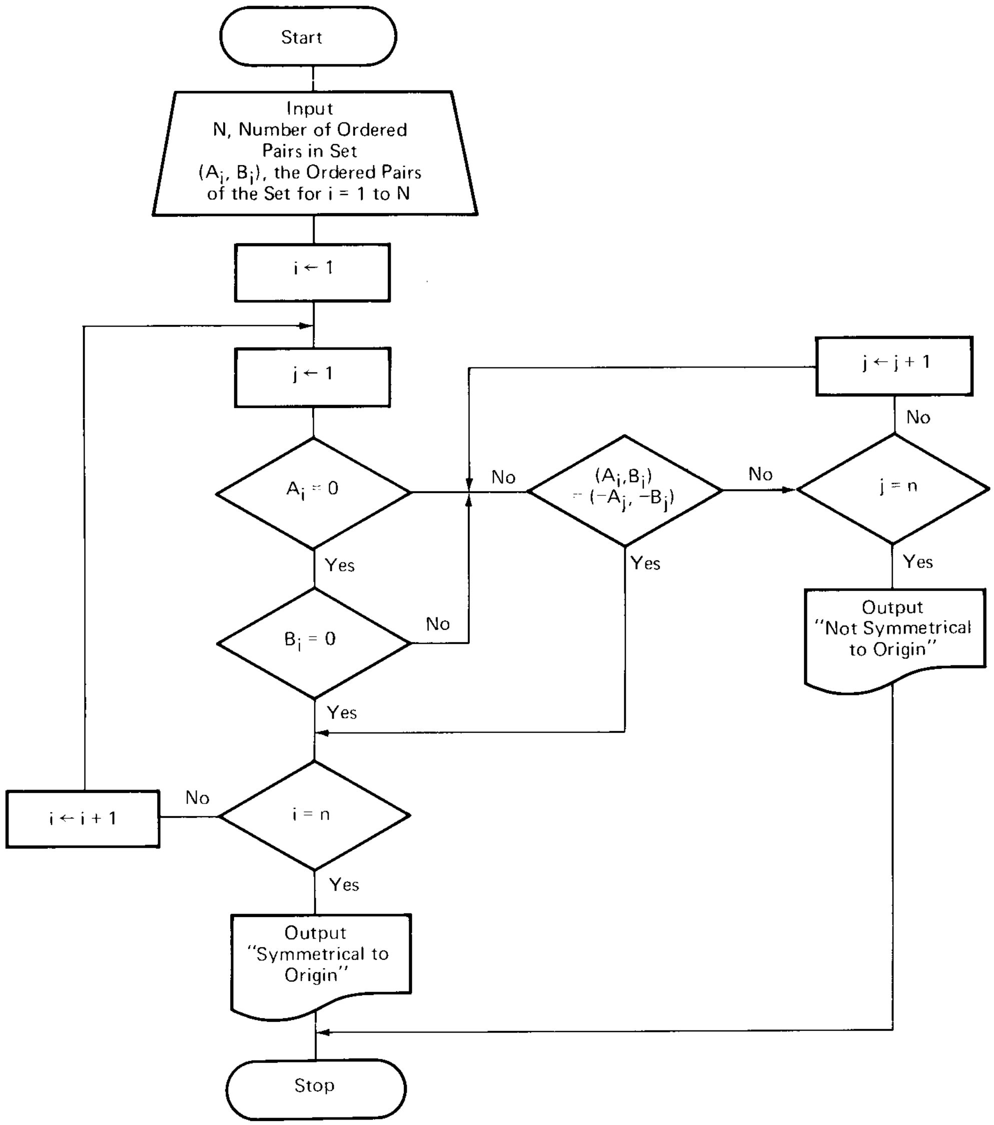
(a) This problem is solved by checking to see if there is a point of symmetry (a, −b) for each ordered pair (a, b). If we find one point which has no point in symmetry, the set is not symmetrical to the x-axis.

(b) Use the same process as in (a), checking for a point of symmetry (−a −b) for each ordered pair (a, b).

Flow Chart
Exercise 6(a)



Flow Chart
Exercise 6(b)



SUGGESTED REFERENCES FOR THIS SECTION

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USING INFINITE FUNCTIONS TO INVESTIGATE OTHER FUNCTION PROPERTIES

The first section of this book discussed function properties that were most easily illustrated by applying them to finite functions. This section presents other properties that are more meaningful when applied to infinite functions defined by polynomials, rational expressions, and other algebraic expressions. Keep in mind, however, that these properties are still properties of finite functions.

FUNCTION ZEROS

Most types of mathematical functions exhibit what is known as a *zero of the function.* A zero of a function is defined as a value of x such that the value of the function is zero. For example, given the function f, defined by $f(x) = x^2 - 1$, the zeros of the function f are $x = 1$ and $x = -1$, since $f(x) = 0$ when $x = \pm 1$.

The computer is very useful in finding the zeros of functions, especially when the zeros are irrational. If the zeros are irrational, we will accept rational approximations as answers.

Exercise 7 — The Zeros of a Function

Write a computer program that will find the zeros of a function defined by:

(1) $f(x) = 10x^2 - 3x - 1$

(2) $f(x) = x^3 + 3x - 1$

(3) $f(x) = 12x^2 - 7x + 1$

Problem Analysis

One approach to this problem is to sketch a graph of the function $f(x) = 10x^2 - 3x - 1$ (see Figure 1). The zeros appear to be between $x = -1$ and $x = 0$, and between $x = 0$ and $x = 1$. To determine the zeros precisely, we will use what we call a root search technique, where we evaluate f in increments of .1 from $x = -1$ to $x = 1$.

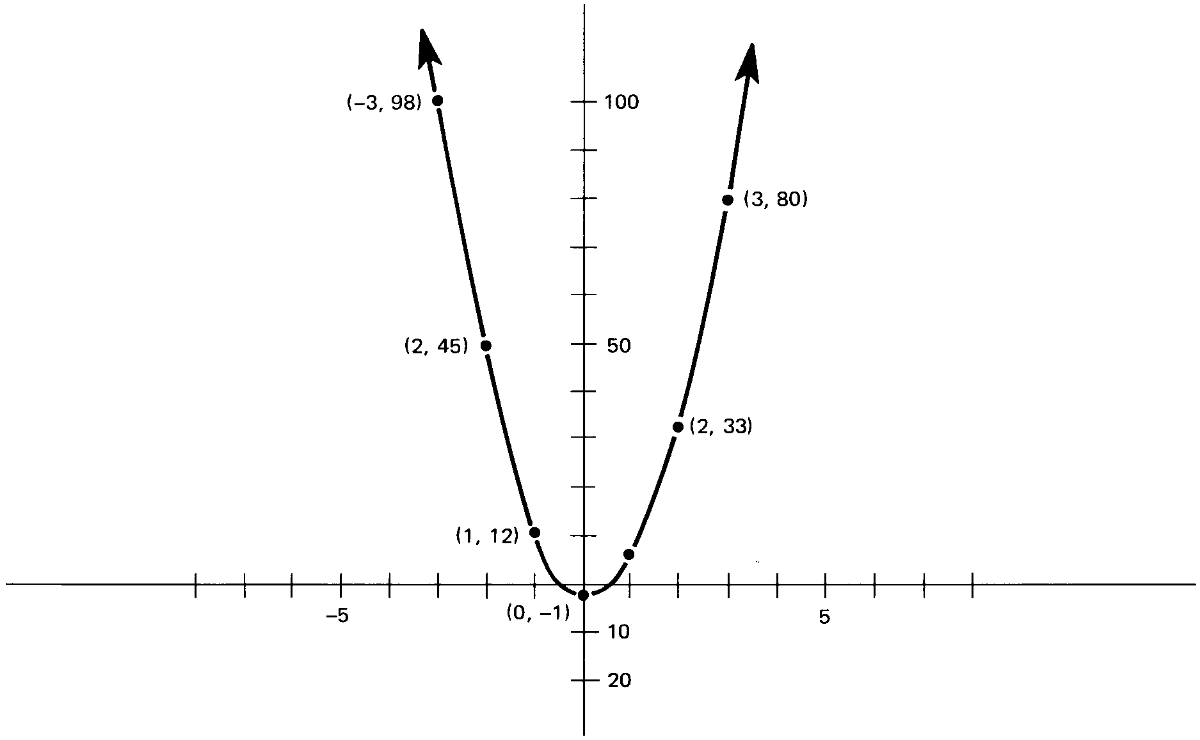


Figure 1. Graph of $f(x) = 10x^2 - 3x - 1$

On an enlarged sketch of a portion of the graph it appears that the zeros are near $x = -.2$ and $x = .4$

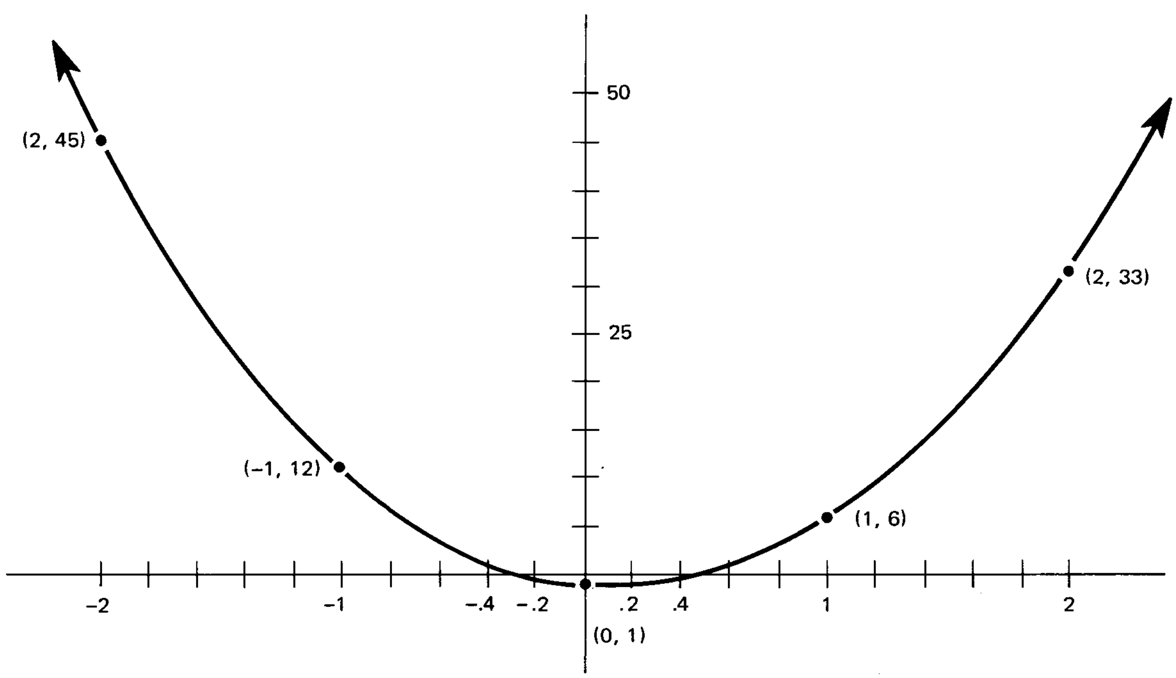


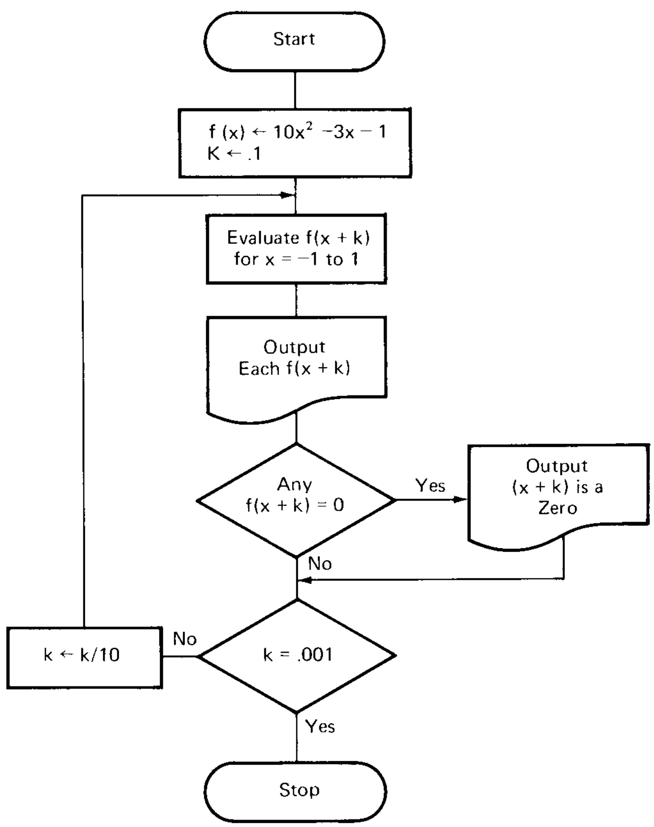
Figure 2. An Enlarged Section of $f(x) = 10x^2 - 3x - 1$

If evaluating f(x) for x = −1, −.9, −.8 . . . 0, .1, .2 . . . 1 does not yield a zero, we can reduce the increment to .01 and evaluate f(x) for x = −1, −.99, −.98, ... 0 . . . . . . .98, .99, 1. If f(x) has rational zeros we should eventually find them by continuing to reduce the increment until we find the precise zero. Luckily, the computer can quickly perform this repetitious procedure for you.

If, however, f(x) has irrational zeros, we can use this same method to estimate them to any degree of precision desired within the capabilities of the computer being used.

For this exercise, let .001 be the precision of tolerance desired. Have the computer print out successive values of the function so that the intervals in which the value of the function changes sign can be observed. For instance, on the enlarged graph above, it appears that $f(-.2) > 0$ and $f(-.1) < 0$ which, if the graph is accurate, indicates a zero between x = −.2 and x = −.1. If we use a tolerance of .001, then we can bracket a zero within a .001 interval.

Flow Chart
Exercise 7



CONTINUITY

We will define a function, f, to be *continuous* if f(x) is an element of the set of real numbers for each real x. For example, a polynomial function $f(x) = A_0 x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_{n-1} x$ is continuous for every real value of x. On the other hand, the function defined by $g(x) = 1/(x^2 - 1)$ is *discontinuous* because for $x = \pm 1$, g(x) is not defined in the set of real numbers. The function g(x) is called a rational function (a rational function is defined to be the quotient of two polynomials.) Figure 3 is a graph of g(x).

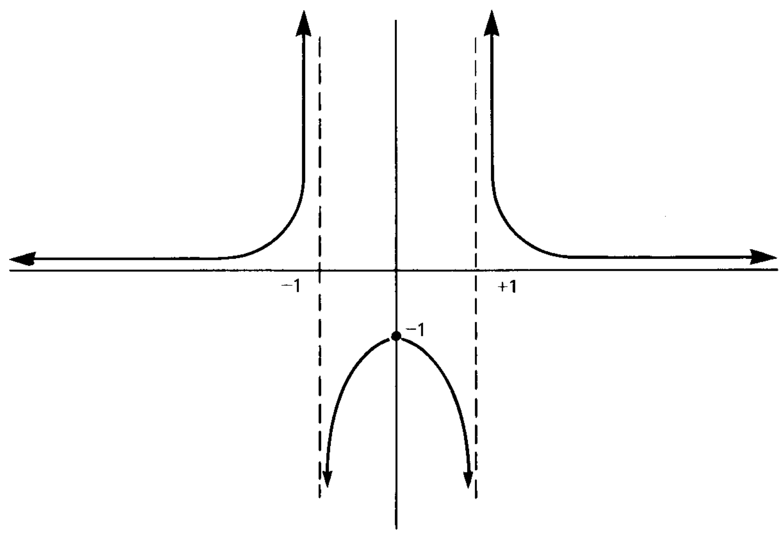


Figure 3. Graph of $g(x) = \dfrac{1}{x^2 - 1}$

Exercise 8 — Discontinuous Functions

(a) Write a program to find the points of discontinuity of the following functions:

$$g(x) = \frac{1}{x^2 - .2x}$$

$$g(x) = \frac{x - .25}{x^2 - .0625}$$

$$g(x) = \frac{x + 1}{10x^2 + 3x - 10}$$

(b) Determine without the use of the computer if the two equations $f(x) = (x-1)/[(x-1)(x+3)]$ and $g(x) = 1/(x+3)$ define the same function.

Problem Analysis

(a) The functions in this exercise are discontinuous when the denominators are equal to zero. Therefore, our task is to determine what values of x cause the denominator of each function to equal zero. In other words, the points of discontinuity occur at the zeros of the function expressed in the denominator. For example, in (a) we need to use the program written for Exercise 7 to find the zeros of $f(x) = x^2 - .2x$. Change the output statement to read "the function is discontinuous at" x.

(b) Since x–1/[(x–1)(x+3)] reduces to 1/(x+3), it would appear that the two equations define the same function. To verify or disprove this assumption, determine where there are points of discontinuity for each function, i.e., for what values of x the denominator of each equation equals zero. If all points of discontinuity are common to both equations, the two equations define the same function.

RELATIVE MAXIMUM AND MINIMUM

Another useful concept in the study of functions is the idea of a *relative maximum.* Figure 4 illustrates this concept. In terms of this illustration, we will define $f(x_0)$ to be a relative maximum of f over the interval from x_i to x_j if and only if $f(x_0) > f(x)$ for all values of x where $x_i \leq x \leq x_j$. A *relative minimum* can be defined in a similar way.

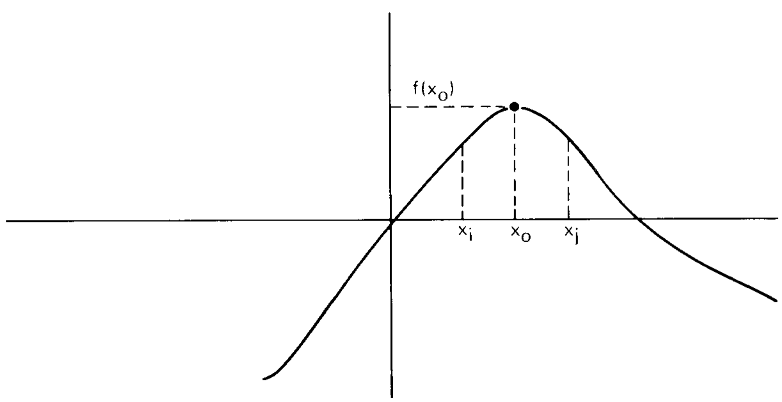


Figure 4. Relative Maximum Value $f(x_o)$ for $x_i \leq x \leq x_j$

For example, consider the function defined by $f(x) = -3x^2 - 2x + 3$. Let's plot some arbitrary values:

x	f(x)
−2	−5
−1	2
0	3
1	−2

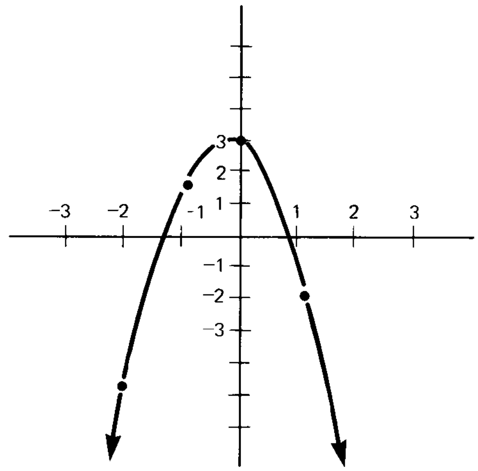


Figure 5. Graph of $f(x) = -3x^2 - 2x + 3$

Figure 5 shows us that the relative maximum value occurs between $x = -1$ and $x = 1$. Figure 6 is an enlargement of this section of the graph.

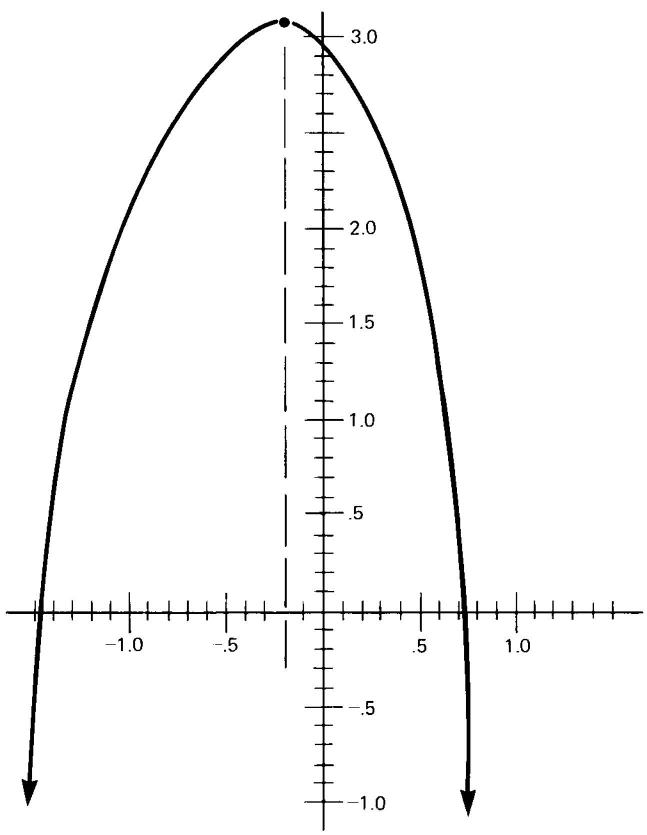


Figure 6. Enlarged Section from Figure 5

To determine the maximum value of f, we will evaluate f starting at $x = -.5$ in increments of $D = .1$ until for some $n \epsilon I \geq 0$, $f(-.5 + nD) \geq f(-.5 + (n+1) D)$. This will indicate a change in direction of the curve between $x = (-.5 + nD)$ and $x = (-.5 + (n+1) D)$.

By use of a calculator we could make the following table.

D = .1	n	x = −.5 + nD	f(x)
	0	−.5	3.25
	1	−.4	3.32
	2	−.3	3.33
	3	−.2	3.28

The table shows that on the interval being considered, the relative maximum value occurs between $x = -.4$ and $x = -.2$.

We will attempt a more precise evaluation of the relative maximum value by letting $D = .01$ and performing evaluation in increments of D on the interval $[-.4, -.2]$.

D = .01	n	x = −.4 + nD	f(x)
	0	−.4	3.32
	1	−.39	3.3237
	2	−.38	3.3268
	3	−.37	3.3293
	4	−.36	3.3312
	5	−.35	3.3325
	6	−.34	3.3332
	7	−.33	3.3333
	8	−.32	3.3328

This table shows that the maximum value $f(x_0) > f(-.34)$ and $f(x_0) > f(-.32)$. You might begin to suspect that $f(x_0) = 3.3333$. Repeating the algorithm of the above table for D/10 we could estimate the relative maximum value to any desired precision.

Exercise 9 — Relative Maximum Value

(a) *Write a computer program using the algorithm illustrated above to find the relative maximum value over a given interval of a function. The output should give the interval of $f(x_0)$ for $D = .1$, $.01$ and $.001$. Apply your program to $f(x) = x^3 - 2x^2 - x + 2$.*

(b) *Apply your program to the same function for $-1 \leq x \leq 0$.*

(c) *A plastics firm finds that it has a surplus of 9" X 12" plastic sheets. It has a market for small plastic trays. The shop foreman is instructed to construct trays with a maximum volume from the sheets in stock. This is done by cutting squares out of the corners and turning up the sides to form the tray as illustrated.*

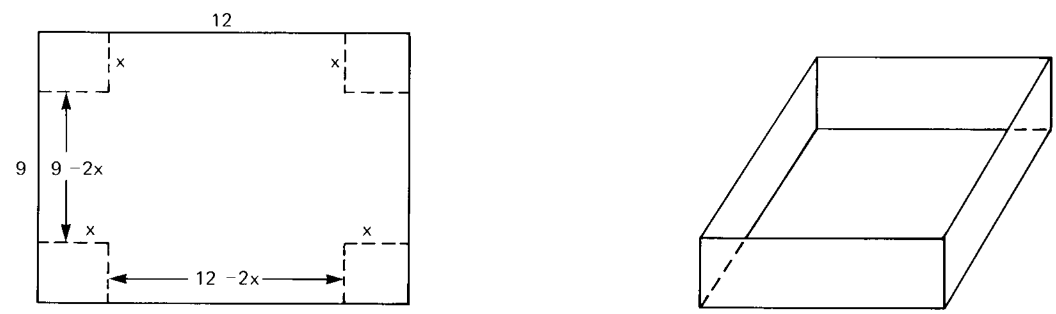


Figure 7 Dimensions of the Plastic Sheets

What would be the dimension of the corners to be cut out to form a tray of maximum volume?

Problem Analysis

(a) Sketch the graph of f(x). Calculate f(0), f(1), f(2), f(3) and f(4). From this you should be able to determine the lower bound A and the upper bound B of the interval in which the maximum value of f(x) occurs. Write the program as outlined by the flow chart on page 28.

(b) This part of the problem is easily solved by substituting the given upper and lower bounds for A and B.

(c) In Figure 7, we see that the dimensions of the tray would be L = (12–2x), W = (9–2x), H = x. The volume of the tray is therefore:

$$\begin{aligned} V &= (12-2x)\,(9-2x)\,x \\ &= 4x^3 - 42x^2 + 108x \end{aligned}$$

First we make a table of values for x and V.

x	V
0	0
1	70
2	80
3	54
4	16

Next, we sketch a graph using the values from the table.

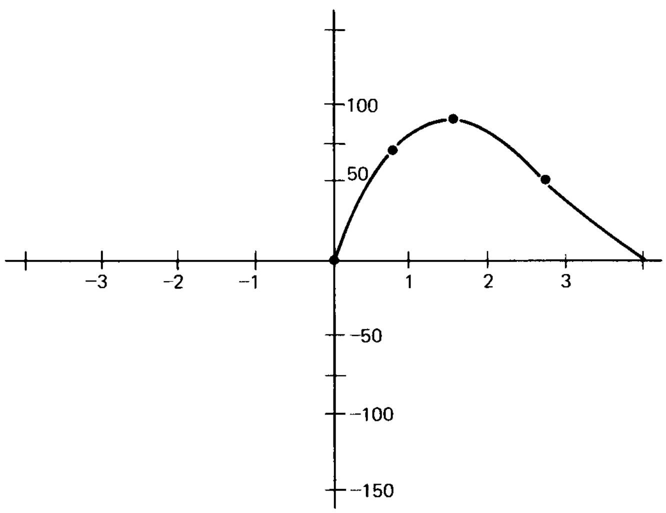
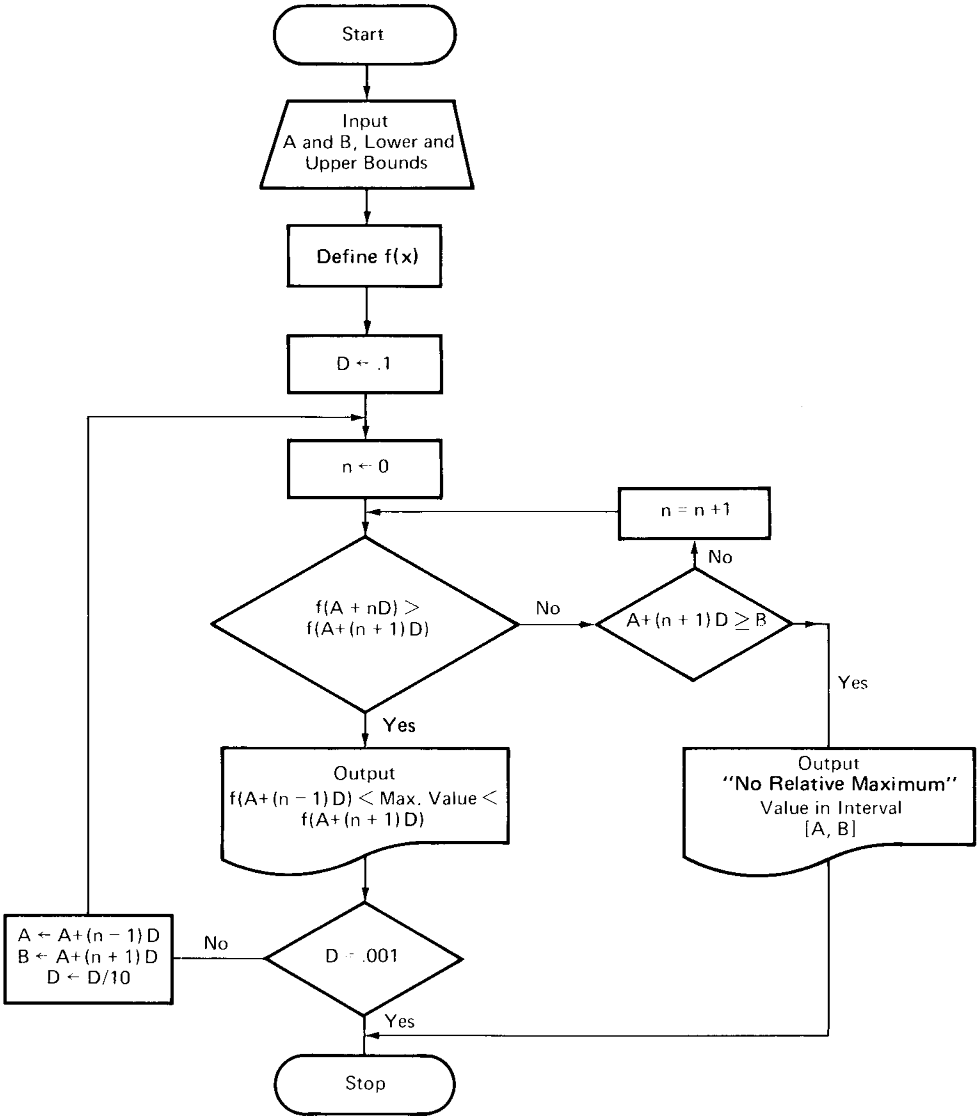


Figure 8. Graph of $V = 4x^3 - 42x^2 + 108x$

From the table and the graph, it appears that the maximum value occurs for some x in the interval $1 \leq x \leq 2$.

Modify your program from part (a) of this exercise to evaluate V over the approximate interval $1 \leq x \leq 2$.

Flow Chart
Exercise 9



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